

Product to Sum Formulas

$$\sin \alpha \sin \beta = \frac{1}{2} [\cos(\alpha - \beta) - \cos(\alpha + \beta)]$$

$$\cos \alpha \cos \beta = \frac{1}{2} [\cos(\alpha - \beta) + \cos(\alpha + \beta)]$$

$$\sin \alpha \cos \beta = \frac{1}{2} [\sin(\alpha + \beta) + \sin(\alpha - \beta)]$$

$$\cos \alpha \sin \beta = \frac{1}{2} [\sin(\alpha + \beta) - \sin(\alpha - \beta)]$$

$$\sin^2 \theta = \frac{1}{2} (1 - \cos(2\theta))$$

$$\cos^2 \theta = \frac{1}{2} (1 + \cos(2\theta))$$

$$\sin(\alpha \pm \beta) = \sin \alpha \cos \beta \pm \cos \alpha \sin \beta$$

$$\cos(\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta$$

Operation	Time Function	Fourier Transform
Linearity	$af_1(t) + bf_2(t)$	$aF_1(\omega) + bF_2(\omega)$
Time shift	$f(t - t_0)$	$F(\omega)e^{-j\omega t_0}$
Time scaling	$f(at)$	$\frac{1}{ a } F\left(\frac{\omega}{a}\right)$
Time transformation	$f(at - t_0)$	$\frac{1}{ a } F\left(\frac{\omega}{a}\right)e^{-j\omega t_0/a}$
Duality	$F(t)$	$2\pi f(-\omega)$
Frequency shift	$f(t)e^{j\omega_0 t}$	$F(\omega - \omega_0)$
Convolution	$f_1(t) * f_2(t)$	$F_1(\omega)F_2(\omega)$
	$f_1(t)f_2(t)$	$\frac{1}{2\pi} F_1(\omega) * F_2(\omega)$
Differentiation	$\frac{d^n[f(t)]}{dt^n}$	$(j\omega)^n F(\omega)$
	$(-jt)^n f(t)$	$\frac{d^n[F(\omega)]}{d\omega^n}$
Integration	$\int_{-\infty}^t f(\tau) d\tau$	$\frac{1}{j\omega} F(\omega) + \pi F(0)\delta(\omega)$

TABLE 4-2
Fourier Transform Pairs

Pair Number	$x(t)$	$X(f)$	Comments on Derivation
1.	$\Pi\left(\frac{t}{\tau}\right)$	$\tau \operatorname{sinc} \pi f$	Direct evaluation
2.	$2W \operatorname{sinc} 2Wt$	$\Pi\left(\frac{f}{2W}\right)$	Duality with pair 1, Example 4-7
3.	$\Lambda\left(\frac{t}{\tau}\right)$	$\tau \operatorname{sinc}^2 \pi f$	Convolution using pair 1
4.	$\exp(-\alpha t)u(t), \alpha > 0$	$\frac{1}{\alpha + j2\pi f}$	Direct evaluation
5.	$t \exp(-\alpha t)u(t), \alpha > 0$	$\frac{1}{(\alpha + j2\pi f)^2}$	Differentiation of pair 4 with respect to α
6.	$\exp(-\alpha t), \alpha > 0$	$\frac{2\alpha}{\alpha^2 + (2\pi f)^2}$	Direct evaluation
7.	$e^{-\pi(t/\tau)^2}$	$\tau e^{-\pi(f/\tau)^2}$	Direct evaluation
8.	$\delta(t)$	1	Example 4-9
9.	1	$\delta(f)$	Duality with pair 7
10.	$\delta(t - t_0)$	$\exp(-j2\pi f t_0)$	Shift and pair 7
11.	$\exp(j2\pi f_0 t)$	$\delta(f - f_0)$	Duality with pair 9
12.	$\cos 2\pi f_0 t$	$\frac{1}{2}\delta(f - f_0) + \frac{1}{2}\delta(f + f_0)$	Exponential representation of cos and sin and pair 10
13.	$\sin 2\pi f_0 t$	$\frac{1}{2j}\delta(f - f_0) - \frac{1}{2j}\delta(f + f_0)$	
14.	$u(t)$	$(j2\pi f)^{-1} + \frac{1}{2}\delta(f)$	Integration and pair 7
15.	$\operatorname{sgn} t$	$(j\pi f)^{-1}$	Pair 8 and pair 13 with superposition
16.	$\frac{1}{\pi t}$	$-j \operatorname{sgn}(f)$	Duality with pair 14
17.	$\hat{x}(t) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{x(\lambda)}{t - \lambda} d\lambda$	$-j \operatorname{sgn}(f)X(f)$	Convolution and pair 15
18.	$\sum_{m=-\infty}^{\infty} \delta(t - mT_s)$	$f_s \sum_{m=-\infty}^{\infty} \delta(f - mf_s),$ $f_s = T_s^{-1}$	Example 4-10